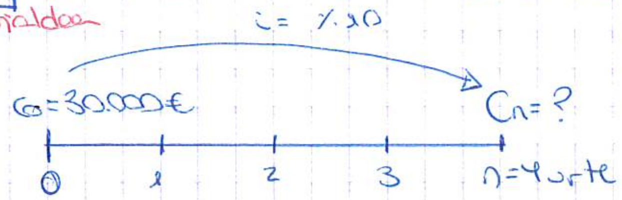
6UD. INTERES KONPOSATUA ETA ERRENTA KONSTANTEAK

1 INTERES ETA KAPITALIZAZIO KONPOSATUA

3. orrialdea

(1)



C_0 : hasierako kapitala = 30.000 €

C_n: bukaerako kapitala = ?

n: denbora = 4 urte

i: interes tasa = %10 → 0,10

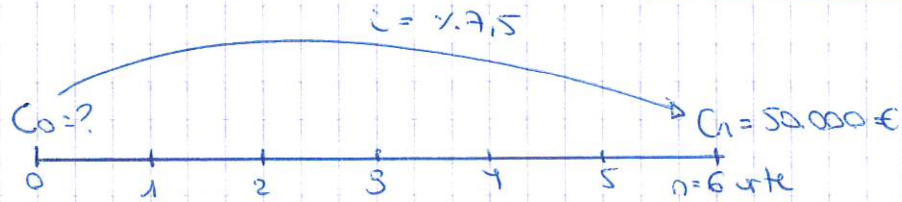
$$\boxed{C_n = C_0 (1+i)^n}$$

$$C_n = 30.000 (1+0,1)^4$$

$$C_n = 30.000 (1,1)^4$$

$C_n = 43.923$ € jasoko du pertsona horrek 4 urteren buruan.

(2)



C_0: hasierako kapitala = ?

C_n: bukaerako kapitala = 50.000 €

n: denbora = 6 urte

i: interes tasa = %7,5 → 0,075

$$C_n = C_0 (1+i)^n$$

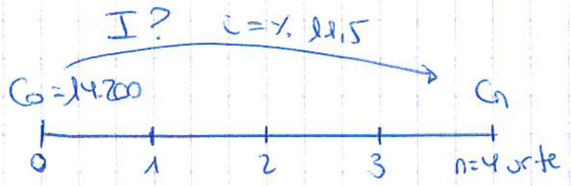
$$\boxed{C_0 = \frac{C_n}{(1+i)^n}}$$

$$\boxed{C_0 = C_n (1+i)^{-n}}$$

$$C_0 = \frac{50.000}{(1+0,075)^6} = \frac{50.000}{(1,075)^6} = $$ 32.398,07592 €-ko hasierako kapitala

$$C_0 = 50.000 (1,075)^{-6} = 32.398,07592€$$

(3)



$$\boxed{I = C_n - C_0}$$

C_0: hasierako kapitala = 14.200 €

C_n: bukaerako kapitala

n: denbora = 4 urte

i: interes tasa = %11,5 → 0,115

I: Interesa

$$C_n = C_0 (1+i)^n$$

$$C_n = 14.200 (1,115)^4 = 21.947,63929 €$$

$$I = C_n - C_0$$

$$I = 21.947,63929 - 14.200 = 7.747,639289 €$$

ordaindu beharko ditu

4

$i = ?$

$C_0 = 900.000€$ $\qquad$ $C_n = 1.400.000€$

0 $\quad$ 1 $\quad$ 2 $\quad$ 3 $\quad$ $n = 4$ urte

$$\boxed{i = \sqrt[n]{\frac{C_n}{C_0}} - 1}$$

C_0: hasierako kapitala $= 900.000€$

C_n: bukaerako kapitala $= 1.400.000€$

n: denbora $= 4$ urte

i: interes tasa $= ?$

$$i = \sqrt[4]{\frac{1.400.000}{900.000}} - 1$$

$$i = 0,11678965 29$$

$\times 100$

$\boxed{i = \%\ 11,6~~7~~896529 \text{-ko interes tasan}}$

5

$i = \%7$

$C_0 = ?$ $\qquad$ $C_n = 2 \cdot C_0$ (Bikoitza)

0 $\qquad$ $n = ?$

C_0: hasierako kapitala

C_n: bukaerako kapitala $= 2C_0$

n = denbora $= ?$

i = interes tasa $= \%7 \Rightarrow 0,07$

C_0 eta C_n zenbatekoa ez dakigunez, asmatu dezakegu.

Adibidez: $C_0 = 100€$ $\quad$ $C_n = 2 \cdot 100 = 200€$

$$\boxed{n = \frac{\log C_n - \log C_n}{\log(1+i)}} \circledast \Longrightarrow n = \frac{\log(200) - \log(100)}{\log(1+0,07)}$$

$$n = \frac{\log(200) - \log(100)}{\log(1,07)}$$

$\circledast$ → Kalkulagailuan log( ) beharrean ln( ) sakatuko dugu.

$n = 10,24$ urte

$\boxed{n = 10 \text{ urte, } 2 \text{ hilabete eta } 27 \text{ egun}}$

$$\left.\begin{array}{l} 1 \text{ urte} \text{ --- } 12 \text{ hilabete} \\ 0,24 \text{ --- } x \end{array}\right\} x = \frac{0,24 \cdot 12}{1} = 2,88 \text{ hilabete}$$

$$\left.\begin{array}{l} 1 \text{ hilabete} \text{ --- } 30 \text{ egun} \\ 0,88 \text{ hilabete} \text{ --- } x \end{array}\right\} x = \frac{0,88 \cdot 30}{1} = 26,4 \text{ egun} \rightarrow 27 \text{ egun}$$

(6)

Co: hasierako kapitala = 415.000 €

Cn: bukaerako kapitala = 1.028.464,46€

n: denbora = ?

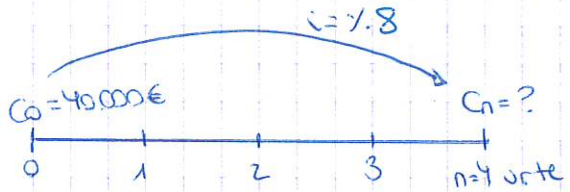
i: interes tasa = %9,5 → 0,095

$$n = \frac{\log C_n - \log C_0}{\log (1+i)}$$

$$n = \frac{\log(1.028.464,46) - \log(415.000)}{\log(1,095)}$$

$$\boxed{n = 10 \text{ urte}}$$

(7)



Co: hasierako kapitala = 40.000 €

Cn: bukaerako kapitala = ?

n: denbora = 4 urte

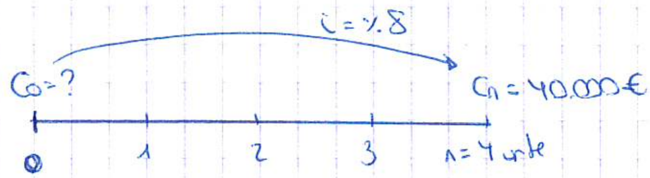
i: interes tasa = %8 → 0,08

$$C_n = C_0 (1+i)^n$$

$$C_n = 40.000 (1,08)^4$$

$\boxed{C_n = 54.419,5584€}$-koa izango da azken kapitala.

(8)



Co: hasierako kapitala

Cn: bukaerako kapitala = 40.000€

n: denbora = 4 urte

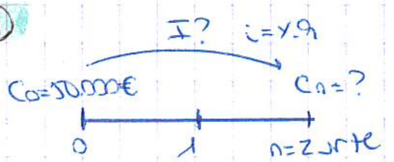
i: interes tasa = %8 → 0,08

$$C_n = C_0 (1+i)^n \;\rightarrow\; C_0 = \frac{C_n}{(1+i)^n} \qquad C_0 = C_n (1+i)^{-n}$$

$$C_0 = C_n (1+i)^{-n} \rightarrow C_0 = 40.000 (1,08)^{-4} = 29.401,1944€$$

$$C_0 = \frac{C_n}{(1+i)^n} = \frac{40.000}{(1,08)^4} = 29.401,1944€$$ jarri beharko ditugu.

(9)



Co: hasierako kapitala = 50.000€

Cn: bukaerako kapitala = ?

n: denbora = 2 urte

i: interes tasa = %9 → 0,09

I: Interesa = ?

$$I = C_n - C_0$$

$$C_n = C_0 (1+i)^n = 50.000 (1,09)^2 = 59.405€$$

$$\rightarrow I = 59.405 - 50.000$$

$\boxed{I = 9.405 € \text{ emango dizkigu mailegu-emaileak}}$

(10)

$i = ?$

$C_0 = 4.000.000€$ — $C_n = 5.000.000€$

0 — 1 — $n = 2$ urte

C_0: hasierako kapitala = 4.000.000€

C_n: bukaerako kapitala = 5.000.000€

n: denbora = 2 urte

i: interes tasa?

$$i = \sqrt[n]{\frac{C_n}{C_0}} - 1$$

$$i = \sqrt[2]{\frac{5.000.000}{4.000.000}} - 1$$

$$i = 0{,}1180339887$$

$\times 100$ → $\boxed{i = \%\ 11{,}80339887\text{-ko interes tasan.}}$

(11)

$i = \%\ 11$

$C_0 = 1.500.000€$ — $C_n = 2.000.000€$

0 — $n = ?$

C_0: hasierako kapitala = 1.500.000€

C_n: bukaerako kapitala = 2000.000€

n: denbora = ?

i: interes tasa = %11 → 0,11

$$n = \frac{\log C_n - \log C_0}{\log(1+i)}$$

$$n = \frac{\log(2.000.000) - \log(1.500.000)}{\log(1{,}11)}$$

$n = 2{,}7566 \rightarrow n = 2{,}76$ urte

$\boxed{n = 2 \text{ urte, 9 hilabete eta 4 egun}}$

1 urte —— 12 hilabete
0,76 —— x

$$x = \frac{0{,}76 \cdot 12}{1} = 9{,}12 \text{ hilabete}$$

1 hilabete —— 30 egun
0,12 —— x

$$x = \frac{0{,}12 \cdot 30}{1} = 3{,}6 \text{ egun} \rightarrow 4 \text{ egun}$$

(12)

%10

$C_0 = ?$ — $C_n = 2 \cdot C_0$

0 — $n = ?$

C_0: hasierako kapitala

C_n: bukaerako kapitala = $2C_0$

n: denbora = ?

i: interes tasa = %10 = 0,10

C_0 eta C_n-en zenbatekoa ez dakigunez, asmatu dezakegu.

Adibidez: $C_0 = 100€$ $C_n = 200€$

$$n = \frac{\log C_n - \log C_0}{\log(1+i)}$$

$$n = \frac{\log(200) - \log(100)}{\log(1{,}10)} = 7{,}272540897 \text{ urte}$$

1 urte —— 12 hil.
0,2725 —— x

$$x = \frac{0{,}2725 \cdot 12}{1} = 3{,}27 \text{ hil}$$

1 hil —— 30 egun
0,27 —— x

$$x = \frac{0{,}27 \cdot 30}{1} = 8{,}1 \text{ egun} \rightarrow 9 \text{ egun}$$

$\boxed{n = 7 \text{ urte, 3 hilabete eta 9 egun.}}$

4. ariketa

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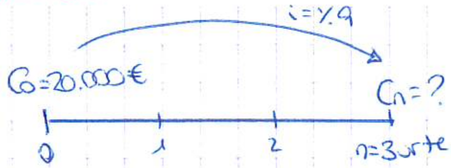

C_0: hasierako kapitala = 20.000€

C_n: bukaerako kapitala = ?

n: denbora = 3 urte

i: interes tasa = %9 → 0,09

$$C_n = C_0 (1+i)^n$$

$$C_n = 20.000\ (1,09)^3$$

$\boxed{C_n = 25.900,58€ \text{ jasoko ditu.}}$

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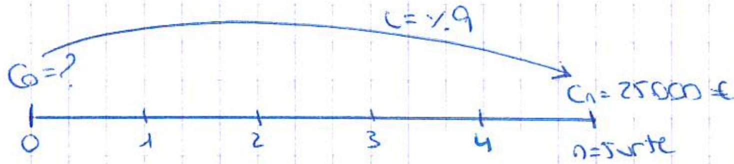

C_0: hasierako kapitala = ?

C_n: bukaerako kapitala = 25.000€

n: denbora = 5 urte

i: interes tasa = %9 → 0,09

$$C_n = C_0 (1+i)^n \rightarrow C_0 = \frac{C_n}{(1+i)^n}$$

$$C_n = C_0 (1+i)^n \rightarrow C_0 = C_n (1+i)^{-n}$$

$$C_0 = 25.000 (1,09)^{-5}$$

$\boxed{C_0 = 16.248,28466€ \text{ jarri genituen.}}$

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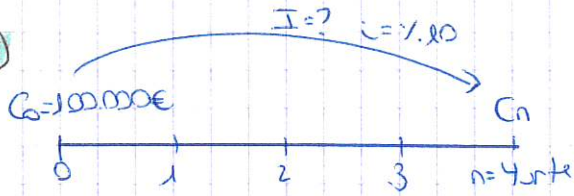

C_0: hasierako kapitala = 100.000€

C_n: bukaerako kapitala

n: denbora = 4 urte

i: interes tasa = %10 → 0,10

I: Interesa (interes totala) = ?

$$I = C_n - C_0$$

$$C_n = C_0 (1+i)^n$$

$$C_n = 100.000\ (1,10)^4 = 146.410€$$

$$I = 146.410 - 100.000$$

$\boxed{I = 46.410€ \text{-ko interesak ordainduko ditu.}}$

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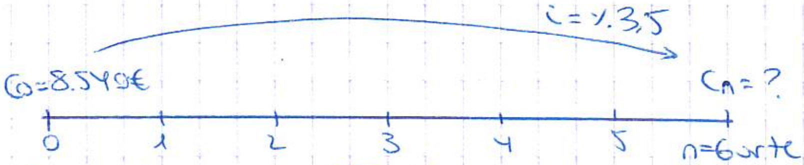

C_0: hasierako kapitala = 8.540€

C_n: bukaerako kapitala = ?

n: denbora = 6 urte

i: interes tasa = %3,5 → 0,035

$$C_n = C_0 (1+i)^n$$

$$C_n = 8.540 (1,035)^6$$

$\boxed{C_n = 10.497,84049€ \text{ edukiko dugu.}}$

(17)

$I = ?$ $i = \%3{,}5$

$C_0 = 1800€$ → C_n

0 1 2 3 4 $n = 5$ urte

C_0: hasierako kapitala $= 1.800€$

C_n: bukaerako kapitala $= ?$

n: denbora $= 5$ urte

i: interes tasa $= \%3{,}5 \rightarrow 0{,}035$

I: interes totala $= ?$

$$I = C_n - C_0$$

$C_n \to ?$, $C_0 \to 1.800€$

$$C_n = C_0 (1+i)^n$$

$$C_n = 1.800 (1{,}035)^5 = 2.137{,}83535€$$

$\boxed{I = 2137{,}83535 - 1800 = 337{,}8353502€}$-ko interesak sortuko ditu.

(18)

$i = ?$

$C_0 = 17.242{,}12€$ → $C_n = 25.829{,}94€$

0 1 $n = 2$ urte

C_0: hasierako kapitala

C_n: bukaerako kapitala

n: denbora $= 2$ urte

i: interes tasa $= ?$

$$i = \sqrt[n]{\frac{C_n}{C_0}} - 1$$

$$i = \sqrt[2]{\frac{25.829{,}94}{17.242{,}12}} - 1$$

$$i = 0{,}2239575821$$

$\cdot 100$ → $\boxed{i = \%22{,}39575821\text{-ko interes tasa konposatuan}}$

(19)

$i = \%7$

$C_0 = 75.000€$ → $C_n = 90.000€$

0 $n = ?$

C_0: hasierako kapitala $= 75.000€$

C_n: bukaerako kapitala $= 90.000€$

n: denbora $= ?$

i: interes tasa $= \%7 \rightarrow 0{,}07$

$$n = \frac{\log C_n - \log C_0}{\log (1+i)}$$

$$n = \frac{\log (90.000) - \log (75.000)}{\log (1{,}07)}$$

$$n = 2{,}694726556 \text{ urte}$$

1 urte —— 12 hil
0,694726556 —— x

$$x = \frac{0{,}694726556 \cdot 12}{1} = 8{,}336718867 \text{ hil}$$

$\boxed{n = 2 \text{ urte, } 8 \text{ hilabete eta } 11 \text{ egun.}}$

1 hilabete —— 30 egun
0,336718867 —— x

$$x = \frac{0{,}336718867 \cdot 30}{1} = 10{,}10156021 \text{ egun}$$

↓

11 egun.

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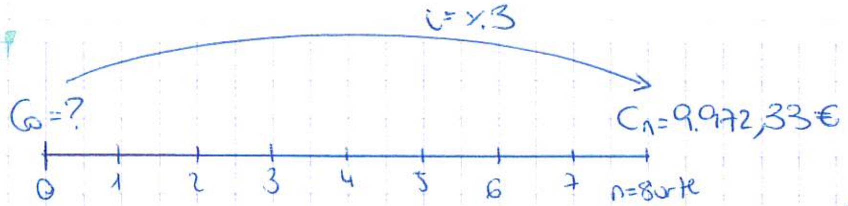

C_0: hasierako kapitala

C_n: bukaerako kapitala

n: denbora = 8 urte

i: interes tasa = $\%3 \rightarrow 0{,}03$

$$C_n = C_0 (1+i)^n \rightarrow C_0 = \frac{C_n}{(1+i)^n} \rightarrow C_0 = C_n (1+i)^{-n}$$

$$C_0 = 9.972{,}33 \, (1{,}03)^{-8}$$

$\boxed{C_0 = 7.872{,}24939€ \text{ jarri zuen.}}$

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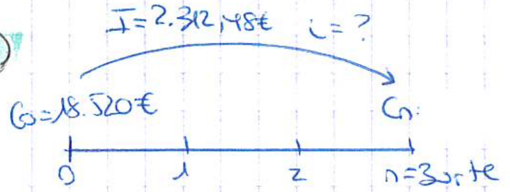

$$i = \sqrt[n]{\frac{C_n}{C_0}} - 1 \quad C_n \rightarrow ?$$

C_0: hasierako kapitala = 18.520 €

C_n: bukaerako kapitala = 2.312,48 €

n: denbora = 3 urte

i: interes tasa = ?

I: Interes totala = 2.312,48 €

$$C_n = C_0 + I = 18.520 + 2.312{,}48 = 20.832{,}48€$$

$$i = \sqrt[3]{\frac{20.832{,}48}{18.520}} - 1$$

$$i = 0{,}0399999787$$

x100

$\boxed{i = \%3{,}99999787 \simeq \%4}$-ko interes tasa konposatuan jarri genuen kapitala.

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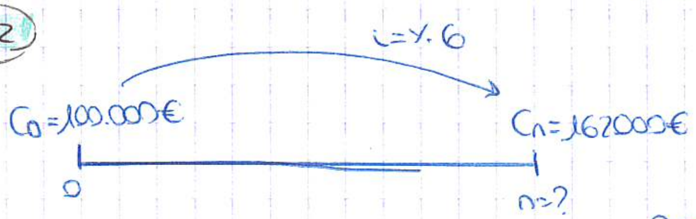

C_0: hasierako kapitala = 100.000 €

C_n: bukaerako kapitala = 162.000€

n: denbora = ?

i: interes tasa = $\%6 \rightarrow 0{,}06$

$$n = \frac{\log C_n - \log C_0}{\log(1+i)}$$

$$n = \frac{\log(162.000) - \log(100.000)}{\log(1+0{,}06)}$$

$$n = 8{,}27931 \text{ urte}$$

$\boxed{n = 8 \text{ urte, 3 hilabete eta 11 egun.}}$

1 urte —— 12 hilabete
0,27931 —— x

$$x = \frac{0{,}27931 \cdot 12}{1} = 3{,}35172 \text{ hilabete}$$

1 hil. —— 30 egun
0,35172 —— x

$$x = \frac{0{,}35172 \cdot 30}{1} = 10{,}5516 \text{ egun} \simeq 11 \text{ egun}$$

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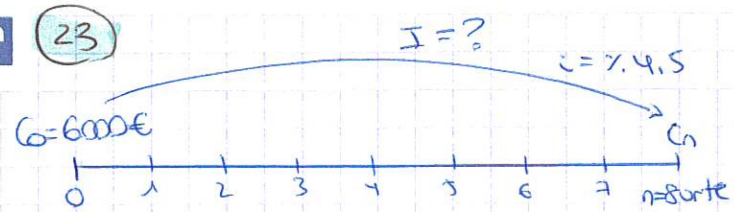

C_0: hasierako kapitala = 6.000€

C_n: bukaerako kapitala = ?

n: denbora = 8urte

i: interes tasa = %4,5 → 0,045

I: Interes totala = ?

$I = C_n - C_0$ (C_n → ?, C_0 → 6000€)

$C_n = C_0 (1+i)^n$

$C_n = 6.000(1,045)^8 = 8.532,603677€$

$\boxed{I = 8.532,603677 - 6.000 = 2.532,603677€\text{-ko}}$ interesak lortuko ditugu.

9.orrialdea

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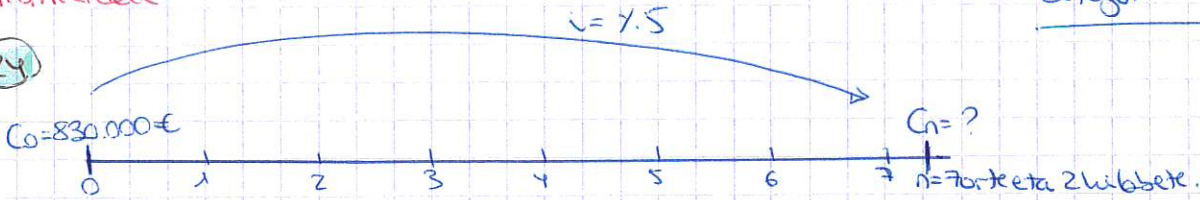


C_0: hasierako kapitala = 830.000€

C_n: bukaerako kapitala = ?

n: denbora = 7urte eta 2 hilabete

i: interes tasa = %5 → 0,05

(*) C_n kalkulatu ahal izateko interes-tasa eta denbora NEURRI BEREAN egon behar dira.

⇓

2 AUKERA DITUGU:

1. AUKERA:

n = 7urte eta 2 hilabete → urtera pasatu ⟹ $n = 7 + 0,16667$

$\boxed{n = 7,16667 \text{ urte}}$

12 hilabete — 1 urte
2 hilabete — x

$\left.\right\} x = \frac{2 \cdot 1}{12} = 0,16667$

$\boxed{C_n = C_0(1+i)^n \Rightarrow C_n = 830.000(1,05)^{7,16667} = 1.177.429,212\ €}$

2. AUKERA:

n = 7urte eta 2 hilabete → hilabetera pasatu ⟹ $n = \overbrace{7 \cdot 12}^{84} + 2 = 86$ hilabete

i = %5 urtekoa → i_{12}-ra pasatu. → $i_k = \sqrt[k]{(1+i)} - 1 \Rightarrow i_{12} = \sqrt[12]{1,05} - 1 = 0,004074123$

$C_n = C_0 (1 + i_k)^{nk}$

$\boxed{C_n = 830.000(1,004074123)^{86} = 1.177.428,941€}$

(25)

$$C_0 = 17.487{,}25€ \xrightarrow{i_4 = \%2} C_n = ?$$

0 — 1 — n = 2 urte

C_0: hasierako kapitala = 17.487,25€

C_n: bukaerako kapitala = ?

n: denbora = 2 urte

i_4: hiruhileko interes tasa = %2 → 0,02

BI AUKERA DITUGU:

1. AUKERA:

n = 2 urte → hiruhilekoetara pasatu → n = 2·4 = 8 hiruhileko

$C_n = C_0 (1+i_k)^{nk} \rightarrow C_n = C_0 (1+i_4)^{n4}$

$C_n = 17.487{,}25 (1{,}02)^8$

$\boxed{C_n = 20.489{,}10051 €}$

2. AUKERA

i_4 → urtekora pasatu → $i = (1+i_k)^k - 1 = (1+0{,}02)^4 - 1 = 0{,}08243216$

$C_n = C_0 (1+i)^n$

$\boxed{C_n = 17.487{,}25 (1{,}08243216)^2 = 20.489{,}10051 €}$

(26)

(A) $C_0 = 10.000€ \xrightarrow{i_4 = \%3} C_n = ?$; 0 — n = 1 urte

(B) $C_0 = 10000 € \xrightarrow{i = \%12{,}5} C_n = ?$; 0 — n = 1 urte

C_0: hasierako kapitala = 10.000 €

C_n = Bukaerako kapitala = ?

n: denbora = 1 urte

i: interes tasa
- hiruhileko interes tasa (A) → i_4 = %3 → 0,03
- urteko interes tasa (B) → i = %12,5 → 0,125

* Denbora eta hasierako kapitala bi kasuetan berdinak direnez. Aukera interesgarriena interes-mota altuena eskaintzen duena da.

(A) i_4 = %3 (hiruhilekoan)
(B) i = %12,5 (urtekoan)

Horrela ezin da kalkulatu.

⇒

Ⓐ Hiruhilekoa $(i_4) \rightarrow$ urtera (i) pasatu $\Rightarrow i = (1+i_k)^k - 1$

$i = (1+i_4)^4 - 1 = (1+0{,}03)^4 - 1 = 0{,}1255088 1 \rightarrow i = \%12{,}550881$

Ⓑ $i = \%12{,}5$

$i_{Ⓐ} = \%12{,}550881 > i_{Ⓑ} = \%12{,}5$
$(i_4 = \%3 > i = \%12{,}5)$

$\rightarrow$ Ⓐ altuagoa da, hau da, interes altuena, beraz $i_4 = \%3$-ko interesa da interesgarriena.

(27)

$i_4 = \%2{,}5 \rightarrow i = ?$

$i = (1+i_k)^k - 1$

$i = (1+i_4)^4 - 1$

$i = (1+0{,}025)^4 - 1 = 1{,}025^4 - 1 = 0{,}1038128906$

$\boxed{i = \%10{,}38128906}$ (urteko efektiboa)

(28)

$i = \%16 \rightarrow i_4 = ?$

$i_k = \sqrt[k]{1+i} - 1$

$i_4 = \sqrt[4]{1+i} - 1$

$i_4 = \sqrt[4]{1+0{,}16} - 1 = \sqrt[4]{1{,}16} - 1 = 0{,}0378019856 5$

$\boxed{i_4 = \%3{,}780198565}$ (hiruhilekoa)

(29)

(interes nominala)

$J_k = J_3 = \%3 \Rightarrow i = ??$ → Ez dago formula zuzenik hau kalkulatzeko.

Dakigunez → $J_k = i_k \cdot k \Rightarrow J_3 = i_3 \cdot 3$

$0{,}03 = i_3 \cdot 3$

$i_3 = \frac{0{,}03}{3} = 0{,}01 \rightarrow i_3 = \%1$

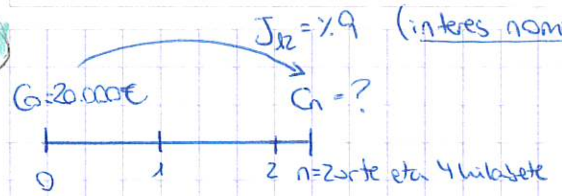
Dakigunez $\Rightarrow i = (1+i_k)^k - 1$

$i = (1+i_3)^3 - 1$

$i = (1+0{,}01)^3 - 1 = 1{,}01^3 - 1 = 0{,}030301$

$\boxed{i = \%3{,}0301}$

30)



C_0: hasierako kapitala = 20.000 €

C_n: bukaerako kapitala = ?

n: denbora = 2 urte eta 4 hilabete $\Rightarrow$ $n = \overbrace{2 \cdot 12}^{24}$ hilabete + 4 hilabete = 28 hil.

J_{12}: Interes nominala hileroko kapitalizazioarekin = %9 → 0,09

$$C_n = C_0 (1 + i_k)^{nk}$$

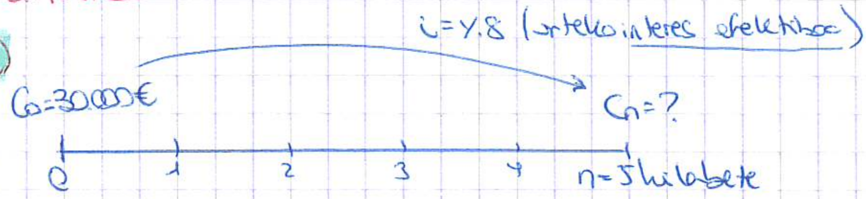
? → $J_k = i_k \cdot k$ → $J_{12} = i_{12} \cdot 12$

$$i_{12} = \frac{J_{12}}{12} = \frac{0,09}{12} = 0,0075 \text{ hileko.}$$

$\boxed{C_n = 20.000 (1 + 0,0075)^{28} = 20.000 (1,0075)^{28} = 24.654,23495€}$ izango da amaierako kapitala.

10. orrialdea

31)



C_0: hasierako kapitala = 30.000 €

C_n: bukaerako kapitala = ?

n: denbora = 5 hilabete

i: interes tasa efektiboa = %8 → 0,08

I: interes totala.

kapitalizazioa hileka egiteko:

i → hilera pasatu ⇒ $i_k = (1+i)^{1/k} - 1$ → $i_{12} = (1+i)^{1/12} - 1$

$$i_{12} = (1 + 0,08)^{1/12} - 1 = 1,08^{1/12} - 1 = 0,0064340311$$

$$C_n = C_0 (1+i)^n$$

$\boxed{C_n = 30.000 (1 + 0,0064340311)^5 = 30.977,6037€}$

I = Cn − Co = 30.977,6037 − 30.000 = 977,6037013€

PFEZ = 977,6037013 · 0,18 = 175,9686662€

Jasoko duena = Cn − PFEZ = 30.977,6037 − 175,9686662 = 30.801,63503€

(32)

$$i = \% 7$$

$C_0 = 20.000$ € $C_n = ?$

0   1   2   3 $n = 3$ urte eta 4 hilabete

C_0: hasierako kapitala = 20.000 €

C_n: bukaerako kapitala = ?

n: denbora = 3 urte eta 4 hilabete

i: interes-tasa = % 7 → 0,07

BI AUKERA:

1.AUKERA:

$n = 3$ urte eta 4 hil → urtetara pasatu

12 hilabete —— 1 urte
4 hil —— x

$x = \frac{4 \cdot 1}{12} = 0,333333$ urte. → $n = 3,333333$ urte

$$C_n = C_0 (1+i)^n$$

$$\boxed{C_n = 20.000 (1 + 0,07)^{3,333333} = 25.059,70253 \text{ €}}$$

2.AUKERA

$n = 3$ urte eta 4 hil → hilabeteetara pasatu ⇒ $n = \overset{36}{3 \cdot 12} + 4 = 40$ hilabete

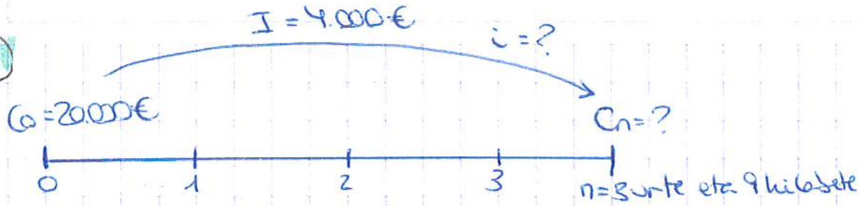
$i = \% 7$ → hilabetetara pasatu → i_{12}

$$i_k = \sqrt[k]{(1+i)} - 1 \rightarrow i_{12} = \sqrt[12]{(1+0,07)} - 1 = 0,005654145387$$

$$C_n = C_0 (1+i)^n$$

$$\boxed{C_n = 20.000 (1 + 0,005654145387)^{40} = 25.059,7031 \text{ €}}$$

33)



C_0: hasierako kapitala = 20.000 €

C_n: bukaerako kapitala = ?

n: denbora = 3 urte eta 9 hilabete

i: interes tasa = ?

I: interes totala = 4.000 €

$C_n = C_0 + I$

$C_n = 20.000 + 4.000 = 24.000$ €

Denbora eta interes-tasa neurri berean egon behar dira.

n = 3 urte eta 9 hilabete ⇒ urtera pasatu:

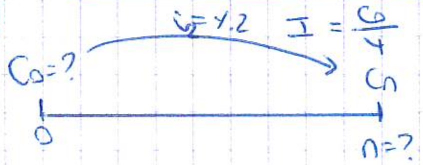
12 hilabete — 1 urte
9 hilabete — x

$$x = \frac{9 \cdot 1}{12} = 0,75 \quad \Rightarrow \quad \underline{n = 3 + 0,75 = 3,75 \text{ urte}}$$

$$i = \sqrt[n]{\frac{C_n}{C_0}} - 1 = \sqrt[3,75]{\frac{24.000}{20.000}} - 1 = 0,04982037889$$

$\boxed{i = \%4,982037889 \text{ -ko interes-tasa efektibo konposatua}}$

34)



C_0: hasierako kapitala
C_n: bukaerako kapitala
n: denbora = ?
i_2: interes tasa seihilekoten → $i_2 = \%2 \to 0,02$
I: Interesa = $\frac{C_0}{4}$

Hasierako kapitala (C_0) eta azken kapitala (C_n) ezagutzen ez dugu, baina bai lortutako interesak $\left(I = \frac{C_0}{4}\right)$.

Asmatu dezakegu hasierako kapitala → $C_0 = 100$ € → $I = \frac{100}{4} = 25$ €

$C_n = C_0 + I = 100 + 25 = 125$ €

$$n = \frac{\log C_n - \log C_0}{\log(1+i)} \Rightarrow n = \frac{\log(125) - \log(100)}{\log(1,02)} = 11,26838111 \text{ seihileko.}$$

2 seihileko → 1 urte
11,26838111 — x

$$x = \frac{11,26838111 \cdot 1}{2} = \underline{5},634190554 \text{ urte}$$

1 urte — 12 hil
0,634190554 — x

$$x = \frac{0,634190555 \cdot 12}{1} = \underline{7},610286648 \text{ hilabete}$$

1 hil — 30 egun
0,610286648 — x

$$x = \frac{0,610286648 \cdot 30}{1} = 18,30859945 \text{ egun} \to \underline{19 \text{ egun}}$$

$\boxed{\to n = 5 \text{ urte, } 7 \text{ hilabete eta } 19 \text{ egun}}$

35

$i = \% 9$

$C_0 = 20.000€$ $\quad C_n = ?$

0 — 1 — 2 $n = 2$ urte eta 4 hilabete

C_0: hasierako kapitala $= 20.000€$

C_n: bukaerako kapitala $= ?$

n: denbora $= 2$ urte eta 4 hilabete

i: interes tasa efektiboa $= \% 9 \rightarrow 0{,}09$

* Interes-tasa eta denbora NEURRI BERDINEAN egon behar dira:

$n = 2$ urte $+ 4$ hilabete $\rightarrow$ urtera pasatu:

$$\left.\begin{array}{l} 12 \text{ hilabete} — 1 \text{ urte} \\ 4 \text{ hilabete} — x \end{array}\right\} x = \frac{4 \cdot 1}{12} = 0{,}333333 \text{ urte} \quad \Big\rangle \; n = 2{,}333333 \text{ urte}$$

$$C_n = C_0 (1+i)^n$$

$$\boxed{C_n = 20.000\,(1+0{,}09)^{2{,}333333}} = 24.454{,}48259€$$ izango da azken kapitala.

36

$i = ?\ j = ?$

$C_0 = 30.000€$ $\quad C_n = ?$

0 — n

a) $J_3 = \% 12 \Rightarrow$ urteko efektibora pasatu (i).

b) $i = \% 12 \Rightarrow$ Horrela geratuko da.

c) $J_{12} = \% 12 \Rightarrow$ urteko efektibora pasatu (i).

Hiru aukera horiek lehenespen ordenaren arabera ordenatu ahal izateko dena urteko efektibora pasatuko dugu:

a) $J_3 = i_3 \cdot 3 \rightarrow i_3 = \frac{J_3}{3} = \frac{0{,}12}{3} = 0{,}04$

$$i = (1+i_k)^k - 1 = (1+0{,}04)^3 - 1 = 0{,}124864 \rightarrow \boxed{i = \% 12{,}4864}$$

b) $\boxed{i = \% 12}$

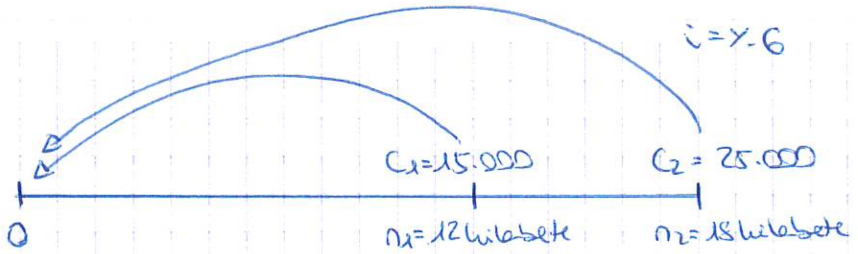
c) $J_{12} = i_{12} \cdot 12 \rightarrow i_{12} = \frac{J_{12}}{12} = \frac{0{,}12}{12} = 0{,}01$

$$i = (1+0{,}01)^{12} - 1 = 0{,}1268250301 \rightarrow \boxed{i = \% 12{,}68250301}$$

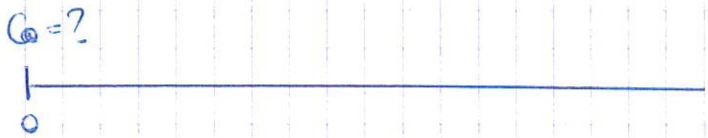
Interesak neurri berean adierazita daudenez, sailkapena interes handienetik interes txikienera izango da, hau da, errentagarritasun handienetik txikienera.

$$\boxed{\begin{array}{l} 1^{\circ} - c)\ i = \% 12{,}68250301 \\ 2^{\circ} - a)\ i = \% 12{,}4864 \\ 3^{\circ} - b)\ i = \% 12 \end{array}}$$

37



C_0: hasierako kapitala
C_n: bukaerako kapitala
n: denbora
i: interes tasa = %6 → 0,06



Kapital horiek baliokideak izateko bete behar da:

$$C_0 = C_1(1+i)^{-n_1} + C_2(1+i)^{-n_2} \qquad C_0 = \frac{C_1}{(1+i)^{n_1}} + \frac{C_2}{(1+i)^{n_2}}$$

* Denbora eta interes-tasa NEURRI BEREAN egon behar dira:

$$i \rightarrow \text{hilabetera pasatu} \rightarrow i_{12} \Rightarrow \begin{cases} i_k = (1+i)^{1/k} - 1 \\ i_k = \sqrt[k]{(1+i)} - 1 \end{cases}$$

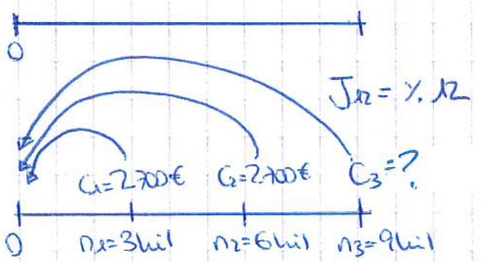
$$i_{12} = \sqrt[12]{(1+0,06)} - 1 = 0,004867550565$$

$$\boxed{C_0 = 15.000(1+0,004867550565)^{-12} + 25000(1+0,004867550565)^{-18} =}$$

$\boxed{= 37.058,62883 \text{ € ordaindu behar du gaur Corte Andaluz enpresak.}}$

11. orrialdea

38 C_0=8000€



C_0: hasierako kapitala
C_n: bukaerako kapitala
n: denbora
J: interes nominala → J_{12} = %12 → 0,12

Kasu honetan 2 aukera horiek baliokideak izateko honako hau bete behar da:

$$C_0 = C_1(1+i)^{-n_1} + C_2(1+i)^{-n_2} + C_3(1+i)^{-n_3} \qquad C_0 = \frac{C_1}{(1+i)^{n_1}} + \frac{C_2}{(1+i)^{n_2}} + \frac{C_3}{(1+i)^{n_3}}$$

* Denborak eta interes-tasak NEURRI BEREAN egon behar dute:

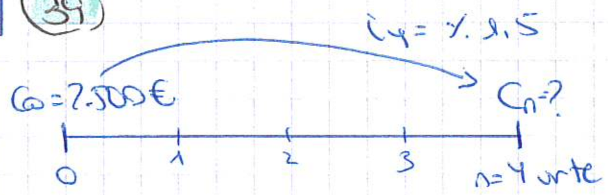
$$J_{12} \rightarrow i_{12} \text{ pasatu} \Rightarrow J_{12} = i_{12} \cdot 12 \rightarrow i_{12} = \frac{J_{12}}{12} = \frac{0,12}{12} = 0,01$$

$$8.000 = 2.700(1+0,01)^{-3} + 2.700(1+0,01)^{-6} + C_3(1+0,01)^{-9}$$

$$8.000 = 5164,115535 + C_3 \cdot 0,9143398242$$

$$\boxed{C_3 = \frac{8.000 - 5164,115535}{0,9143398242}} = \boxed{3.101,565075 \text{ € ordaindu ditu Aliciak 9 hilabetera.}}$$

39



C_0: hasierako kapitala $= 2.500€$

C_n: bukaerako kapitala $= ?$

n: denbora $= 4$ urte

i: interes tasa $\Rightarrow i_4 = \% 1,5 \rightarrow 0,015$

* Denbora eta interes-tasa neurri berean; bi aukera:

① $n = 4$ urte $\rightarrow$ hiruhilekotara pasatu

$n = 4 \cdot 4 = 16$ hiruhileko

$$\left.\begin{array}{l} 1 \text{ urte} \text{ — } 4 \text{ hiruhileko} \\ 4 \text{ urte} \text{ — } x \end{array}\right\} x = \frac{4 \cdot 4}{1} = 16$$

$C_n = C_0 (1+i)^n$

$\boxed{C_n = 2.500 (1,015)^{16} = 3.172,463869€ \text{ edukiko dugu.}}$

② i_4 (hiruhilekoa) $\rightarrow i$ (urtera) pasatu

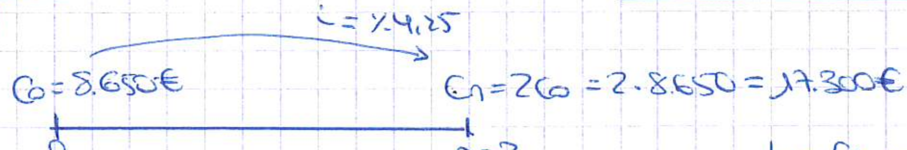
$i = (1+i_k)^k - 1$

$i = (1+i_4)^4 - 1 = (1,015)^4 - 1 = 0,06136355063$

$C_n = C_0 (1+i)^n$

$\boxed{C_n = 2500 (1,06136355063)^4 = 3.172,463869€ \text{ edukiko dugu.}}$

40



C_0: hasierako kapitala $= 8.650€$

C_n: bukaerako kapitala $= 2 \cdot C_0 = 17.300€$

n: denbora $= ?$

i: interes tasa $= \% 4,25 \rightarrow 0,0425$

$$n = \frac{\log C_n - \log C_0}{\log(1+i)}$$

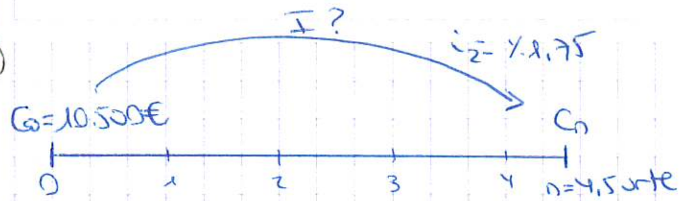
$$n = \frac{\log(17.300) - \log(8.650)}{\log(1,0425)} = 16,65351492$$

$\boxed{n = 16 \text{ urte, } 7 \text{ hilabete eta } 26 \text{ egun.}}$

$$\left.\begin{array}{l} 1 \text{ urte} \text{ — } 12 \text{ hilabete} \\ 0,65351492 \text{ — } x \end{array}\right\} x = \frac{0,65351492 \cdot 12}{1} = 7,84217904 \text{ hilabete}$$

$$\left.\begin{array}{l} 1 \text{ hilabete} \text{ — } 30 \text{ egun} \\ 0,84217904 \text{ — } x \end{array}\right\} x = \frac{0,84217904 \cdot 30}{1} = 25,2653712 \text{ egun} \simeq 26 \text{ egun}$$

41



C_0: hasierako kapitala = 10.500€

C_n: bukaerako kapitala

n: denbora = 4,5 urte

i: interes tasa → $i_2 = \% 1,75 \rightarrow 0,0175$

I: interesa = ? $\Rightarrow$ $I = C_n - C_0$

* Denbora eta interes-tasa neurri berean egon behar dira:

n = 4,5 urte → seihilekora pasatu:

$n = 4,5 \cdot 2 = 9$ seihileko

$$\left.\begin{array}{l} 1 \text{ urte} \;—\; 2 \text{ seihileko} \\ 4,5 \text{ urte} \;—\; x \end{array}\right\} x = \frac{4,5 \cdot 2}{1} = 9$$

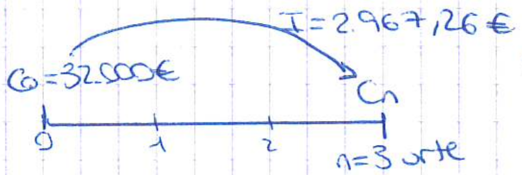
$$C_n = C_0 (1+i)^n$$

$$C_n = 10.500 (1,0175)^9 = 12.274,36575 €$$

$$I = C_n - C_0$$

$$I = 12.274,36575 - 10.500€ \Rightarrow \boxed{I = 1.774,365749 €\text{-ko interesak lortuko ditugu.}}$$

42



$i_4 = ?$

C_0: hasierako kapitala = 32.000€

C_n: bukaerako kapitala = ?

n: denbora = 3 urte

i: interes tasa → $i_4 = ?$

I: Interesa = 2.967,26€

$$i = \sqrt[k]{\frac{C_n}{C_0}} - 1$$

$$I = C_n - C_0 \rightarrow 2.967,26 = C_n - 32.000$$

$$C_n = 2.967,26 + 32.000 = 34.967,26€$$

$$i = \sqrt[3]{\frac{34.967,26}{32.000}} - 1 = 0,0299996073$$

$$i_4 = (1+i)^{1/4} - 1 = \quad // \quad i_4 = \sqrt[4]{(1+i)} - 1 \;(*)$$

$$= (1 + 0,0299996073)^{1/4} - 1 =$$

$$= 0,007416111551$$

$$\boxed{i_4 = \% 0,7416111551 \text{-ko hiruhileko tasa efektiboan jarri genuen dirua.}}$$

(*) $i_4 = \sqrt[4]{(1 + 0,0299996073)} - 1 =$

$= 0,007416111551$

(43)

$i_4 = \% \ 1,6 \rightarrow 0,016$

$i = (1 + i_k)^k - 1$

$i = (1 + 0,016)^4 - 1 = 0,06555244954$

$$\boxed{i = \% \ 6,555244954}$$

(44)

$J_{12} = \% 4$

$C_0 = 10.000€$ $\qquad$ $C_n = 18.664€$

0 $\qquad$ $n = ?$

Eragiketa egin ahal izateko interes-tasa efektiboa kalkulatu behar dugu:

$J_k = i_k \cdot k \Rightarrow i_k = \dfrac{J_k}{k}$

$i_{12} = \dfrac{J_{12}}{12} = \dfrac{0,04}{12} = 0,00333333$

C_0: hasierako kapitala $= 10.000€$

C_n: bukaerako kapitala $= 18.664€$

n: denbora $= ?$

J: interes nominala $\rightarrow J_{12} = \% 4 \rightarrow 0,04$

$$n = \frac{\log C_n - \log C_0}{\log(1+i)} \rightarrow n = \frac{\log(18.664) - \log(10.000)}{\log(1,00333333)} = 187,5154524 \text{ hilabete}$$

12 hilabete —— 1 urte
187,5154524 hilabete —— x

$$x = \frac{187,5154524 \cdot 1}{12} = 15,6262877 \text{ urte}$$

1 urte —— 12 hilabete
0,6262877 urte —— x

$$x = \frac{0,6262877 \cdot 12}{1} = 7,5154524 \text{ hilabete}$$

1 hilabete —— 30 egun
0,5154524 —— x

$$x = \frac{0,515425 \cdot 30}{1} = 15,463572 \text{ egun} \simeq 16 \text{ egun.}$$

$$\boxed{n = 15 \text{ urte, } 7 \text{ hilabete eta } 16 \text{ egun.}}$$

45) 2 AUKERA

$C_{0_{(2)}} = 0{,}2 \cdot C_{0_{(1)}} = 0{,}2 \cdot 250.000 = 50.000€$

1 AUKERA

$C_0 = 250.000 €$

$C_1 = 60.000 €$, $n_1 = 3$ hilabete

$C_2 = 150.000 €$, $n_2 = 2$ urte

$i = \% 10$

2 AUKERA $\rightarrow$ $C_0 + C_1 + C_2 = 260.000 €$

Aukera merkeena 250.000 € esludintan ordaintzea da, beste aukeraren kapitalen batura 260.000€-koa delako eta honi interesak gehitu behar dizkiogu.

12. ariketa

46) $J_4 = \% 6$

$C_0 = 4200 €$ $C_n = ?$

0 1 2 3 4 5 6 $n = 6{,}5$ urte

* Kalkuluak egin ahal izateko interes-tasa efektiboa izan behar du. Gainera denboraren eta interes-tasak NEURRI BERean egon behar dute.

C_0: hasierako kapitala $= 4200 €$

C_n: bukaerako kapitala $= ?$

n: denbora $= 6{,}5$ urte

i: interes tasa

J: interes nominala $\rightarrow J_4 = \% 6 \rightarrow 0{,}06$

$J_k = i_k \cdot k$

$J_4 = i_4 \cdot 4 \rightarrow i_4 = \frac{J_4}{4} = \frac{0{,}06}{4} = 0{,}015$

$i = (1 + i_k)^k - 1 \rightarrow i = (1 + i_4)^4 - 1$

$i = (1 + 0{,}015)^4 - 1 = 0{,}0613635506 3$

$C_n = C_0 (1+i)^n$

$\boxed{C_n = 4.200 (1 + 0{,}06136355063)^{6{,}5} = 6.185{,}38045 €} > 6.000 €$

6 urte eta erdiren buruan 6.000 €-ko ordainketa egin ahal izango dut.

47) $i = \% 3$

$C_0 = ?$ $C_n = 23.185{,}48 €$

0 1 2 3 4 $n = 5$ urte

C_0: hasierako kapitala $= ?$

C_n: bukaerako kapitala $= 23.185{,}48 €$

n: denbora $= 5$ urte

i: interes tasa $= \% 3 \rightarrow 0{,}03$

$C_0 = \frac{C_n}{(1+i)^n} = \frac{23.185{,}48}{(1{,}03)^5} = \boxed{19.999{,}99872 €}$

$C_n = C_0 (1+i)^n$

(berdinak dira)

$\rightarrow C_0 = C_n (1+i)^{-5} = 23.185{,}48 (1{,}03)^{-5} = 19.999{,}99872 €$